[2]Buffinton, K. W., and Kane, T. R., "Dynamics of Beams Moving over Supports," *International Journal of Solids and Structures*, Vol. 21, No. 7, 1985, pp. 617–643.

[3]Tadikonda, S. S. K., "Modeling of Translational Motion Between Two Flexible Bodies Connected via Three Points," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 6, 1995, pp. 1392–1397.

[4]Bhatta, M. G., Subramaniam, M., Venngopal, R., and Neguyen, L. H., "Modeling and Simulation of the Mobile Transporter System of International Space Station," *AIAA/USAF/NASA/ISSMO 7th Symposium on Multidisciplinary Analysis and Optimization*, AIAA, Reston, VA, 1998, pp. 1639–1649.

[5]Sharf, I., "Geometric Stiffening in Multibody Dynamics Formulation," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 4, 1995, pp. 882–890.

[6]Kane, T. R., and Levinson, D. A., *Dynamics: Theory and Applications*, McGraw–Hill, New York, 1985, p. 158.

[7]Thomas, S., and Stubbs, M., "Dynamic Analysis of the Space Station Truss Structure Based on Continuum Representation," *AIAA/ASME/ASCE/AHS/ASC 30th Conference on Structures, Structural Dynamics, and Materials*, AIAA, Washington, DC, 1989, pp. 1062–1068.

[8]Subramaniam, M., Byun, K. W., and Hua, T., "Control Structure Interaction Analysis of the International Space Station," *AIAA/USAF/NASA/ISSMO 7th Symposium on Multidisciplinary Analysis and Optimization*, AIAA, Reston, VA, 1998, pp. 1628–1638.

[9]Carbonneau, R., Dubois, J., and Harris, G., "An Optical Gun Muzzle Sensor to Improve Firing Accuracy," *Optical Testing and Metrology*, Vol. 661, Society of Photo-Optical Instrumentation Engineers, Bellingham, WA, 1986, pp. 352–358.

[10]*Jane's Armor and Artillery 1995–1996*, edited by C. F. Foss, International Thomson Publ., Surrey, England, U.K., 1996, p. 681.

Near-Optimal Deflection of Earth-Approaching Asteroids

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Introduction

THE spectacular collision of the Shoemaker-Levy 9 asteroid with Jupiter in July 1994 was a dramatic reminder that the Earth has and will continue to experience catastrophic impact events. Although the frequency of such massive collisions is very low, smaller objects collide with the Earth regularly and do damage that would be intolerable in any populated region. A consensus is developing that, although the probability for collision is low, the potential for destruction is immense and, thus, some resources should be devoted to threat detection and possible interdiction.

The population of known Earth-crossing asteroids is large and continuously increasing by virtue of discovery. As of late 2000, 275 Earth-crossing asteroids (ECAs) that may be termed potentially hazardous are known.[1] For the prevention of a catastrophic impact, the asteroid must be intercepted, at the earliest possible time, and then deflected or destroyed. One strategy may be to detonate a large nuclear weapon at or near the surface of the asteroid or comet, vaporizing part of the surface and yielding an impulsive velocity change because of the momentum imparted to the ejected mass.[2,3]

In a previous Note this author found optimal, that is, minimum time-of-flight, trajectories for the interception of dangerous asteroids.[4] Low-thrust, high specific impulse propulsion was assumed because of the significant advantages it provides in propulsive mass required for a given mission. (An important conclusion of the previous research was that the much greater efficiency of low-thrust electric propulsion would yield a payload mass for the mission of approximately 12% of launch vehicle mass compared with only about 3% using conventional chemical propulsion.)

In this companion Note, the emphasis will be on the process of optimizing the deflection of the dangerous asteroid, at the time of its close approach to Earth, by a given impulse applied at an earlier time. We will again assume that a collision (or near-collision) is imminent, that is, will occur before the asteroid has made another complete revolution about the sun. For most of the known ECAs this means a period of a few years at most to take some type of preventative action.

If the time between application of the deflection impulse and close approach, Δt, is very brief, the deflection $\bar{\Delta}$ can be approximated assuming rectilinear motion, yielding $\bar{\Delta} = (\Delta\bar{v})\,(\Delta t)$ (Ref. 2). The opposite case, in which the asteroid will make more than one orbit of the sun before its close approach to Earth, is an interesting one for which the relationship between the impulse and resulting deflection is much more complicated. Park and Ross show that for this case the best place to apply a deflecting impulse is at asteroid perihelion.[5] Their analysis is one of very few to optimize explicitly the deflecting impulse, for arbitrary Δt, assuming Keplerian motion. However, their solution is two dimensional, that is, it assumes the asteroid orbits the sun in the ecliptic plane. However, 104 of the 275 known potentially hazardous ECAs have inclination greater than 10 deg, and 49 have inclination greater than 20 deg (Ref. 1).

The intention of this work is to show that a near-optimal determination of the direction in which an impulse should be applied to the asteroid, as well as the resulting deflection, can be found without any explicit optimization. The method is easily applied to the true, three-dimensional geometry of the problem.

Maximization of the Deflection

The objective is to determine the direction in which a given impulse should be applied to the asteroid at interception to maximize the subsequent close-approach distance of the asteroid (to the Earth). At the time of interception, t_0, the system state transition matrix $\Phi(t, t_0)$ determines the perturbation in position $\delta\bar{r}$ and velocity $\delta\bar{v}$, which will result at time t due to a perturbation in position and velocity applied at t_0 (Ref. 6), that is,

$$\begin{bmatrix}\delta\bar{r}\\ \delta\bar{v}\end{bmatrix} = \Phi(t,t_0)\begin{bmatrix}\delta\bar{r}_0\\ \delta\bar{v}_0\end{bmatrix} = \begin{bmatrix}\tilde{R} & R\\ \tilde{V} & V\end{bmatrix}\begin{bmatrix}\delta\bar{r}_0\\ \delta\bar{v}_0\end{bmatrix} \tag{1}$$

Therefore,

$$\delta\bar{r}(t) = [R]\delta\bar{v}_0(t_0) \tag{2}$$

and

$$[R] = (r_0/\mu)(1-F)\big[(\bar{r}-\bar{r}_0)\bar{v}_0{}^T - (\bar{v}-\bar{v}_0)\bar{r}_0{}^T\big] + (C/\mu)\bar{v}\bar{v}_0^T + G[I]$$

$$F = 1-(r/p)(1-\cos\theta), \qquad \cos\theta = \bar{r}\cdot\bar{r}_0/rr_0$$

$$G = (1/\sqrt{\mu})\Big[\big(rr_0/\sqrt{p}\big)\sin\theta\Big] \tag{3}$$

where p is the orbit semilatus rectum,

$$C = (1/\sqrt{\mu})\big[3U_5 - \chi U_4 - \sqrt{\mu}(t-t_0)U_2\big]$$

$$\chi = \sqrt{a}(E-E_0), \qquad \alpha = 1/\mathrm{a}$$

Here a is the orbit semimajor axis, E is the eccentric anomaly, μ is the gravitational parameter, and $U_1(\chi,\alpha)$, $U_2(\chi,\alpha)$, $U_3(\chi,\alpha)$, $U_4(\chi,\alpha)$, and $U_5(\chi,\alpha)$ are the universal functions.[6]

We want to maximize $|\delta\bar{r}(t_c)| = \max([R]\delta\bar{v}_0)$ where the time of interest t_c is the time of close approach to Earth. Equivalently, we may maximize $\delta\bar{v}_0^T[R]^T[R]\delta\bar{v}_0$. This quadratic form is maximized, for given $|\delta\bar{v}_0|$, if $\delta\bar{v}_0$ is chosen parallel to the eigenvector of $[R]^T[R]$ that is conjugate to the largest eigenvalue of $[R]^T[R]$. This yields the optimal direction for the perturbing velocity impulse $\delta\bar{v}_0$, which will be expressed on the space-fixed basis because this is the basis

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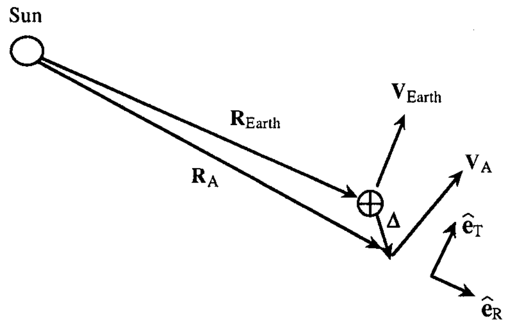

Fig. 1 Position of the Earth and asteroid at the close approach.

on which $[R]$ is implicitly expressed. Then $\delta\bar{v}_0$ may be expressed in an asteroid-fixed radial, transverse, normal basis as

$$\delta\bar{v}_{0\,\text{RTN}} = \begin{bmatrix} c\theta c\Omega - \text{cis}\,\Omega s\theta & c\theta s\Omega + \text{cic}\,\Omega s\theta & \text{sis}\,\theta \\ -s\theta c\Omega - \text{cis}\,\Omega c\theta & -s\theta s\Omega + \text{cic}\,\Omega c\theta & \text{sic}\,\theta \\ \text{sis}\,\Omega & -\text{sic}\,\Omega & \text{ci} \end{bmatrix} \delta\bar{v}_{0XYZ} \tag{4}$$

where $s\theta$ and $c\theta$ are abbreviations for $\sin\theta$ and $\cos\theta$, respectively.

It may be argued that, because in this approach the deflection $|\delta\bar{r}(t)|$ is maximized only at time t_c, which would be the time of collision absent the deflecting impulse, the subsequent motion of the asteroid and Earth may bring the two bodies closer together. That is, if we let

$$\bar{R} = \bar{R}_A - \bar{R}_{\text{Earth}} \tag{5}$$

where subscript A is the asteroid and both position vectors are determined with respect to the sun, as shown in Fig. 1, then the deflection $\bar{\Delta} = \delta\bar{r}(t_c) = \bar{R}(t_c)$. In the analysis of Park and Ross,[5] this is explicitly prevented by including necessary conditions for the nonlinear programming problem; $\bar{R} = 0$ and $\dot{R} \geq 0$ at the time of close approach. The simplicity of the method described here for finding the near-optimal deflection precludes incorporating these necessary conditions; however, it is possible to guarantee that at time t_c the separation of the asteroid and the Earth is increasing. Since

$$\frac{\mathrm{d}\bar{R}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\bar{R}_A - \bar{R}_{\text{Earth}}) = \bar{V}_A - \bar{V}_{\text{Earth}} \tag{6}$$

then

$$\dot{R} = \frac{\mathrm{d}\bar{R}}{\mathrm{d}t} \cdot \frac{\bar{R}}{R} = (\bar{V}_A - \bar{V}_{\text{Earth}}) \cdot \frac{\bar{\Delta}}{\Delta} \tag{7}$$

at time t_c, where $\bar{\Delta} = \delta\bar{r}(t_c) = \bar{R}(t_c)$, as shown in Fig. 1. To have the separation of the bodies increasing, the scalar product in Eq. (7) must be positive. This can always be achieved; recall from Eq. (2) that $\bar{\Delta} = \delta\bar{r}(t) = R\,\delta\bar{v}_0(t_0)$, where $\delta\bar{v}_0$ is chosen parallel to the eigenvector of $[R]^T[R]$ that is conjugate to the largest eigenvalue of $[R]^T[R]$. Thus, the magnitude of the deflection is unaffected by whether the deflection $\delta\bar{v}_0$ or $-\delta\bar{v}_0$ is applied to the asteroid at interception so long as this condition obtains; one of the two choices will cause the asteroid to miss the Earth by $\bar{\Delta}$ and separate farther in their subsequent relative motion.

Examples

As an example, optimization of the deflection impulse has been applied to the case of the Earth-approaching asteroid 1991RB. Optimal, minimum-time-of-flight trajectories to this asteroid, using impulsive thrust for Earth escape followed by continuous low-thrust electric propulsion, were found in a previous paper by this author.[4] The asteroid orbit elements are,[1] as of 1 September 1991, $a = 1.45$ astronomical unit (AU), $e = 0.484$, $i = 19.50$ deg, $\Omega = 359.6$ deg, $\omega = 68.7$ deg, and $M = 320.1$ deg.

This asteroid approached the Earth to within 0.04 AU, or 15 lunar distances, on 19 September 1998. Its orbit semimajor axis, eccentricity, and inclination are all very representative of the asteroids in the catalog of potentially hazardous objects.[1]

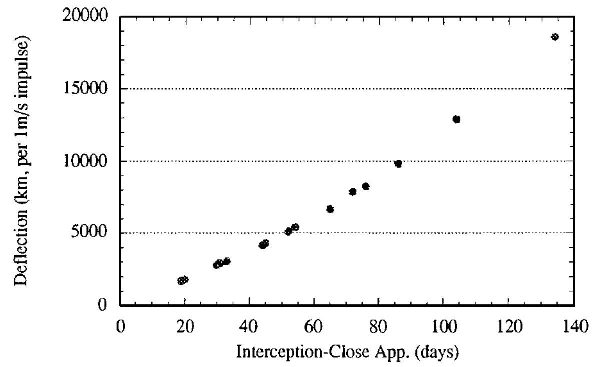

Fig. 2 Specific deflection as a function of date of interception.

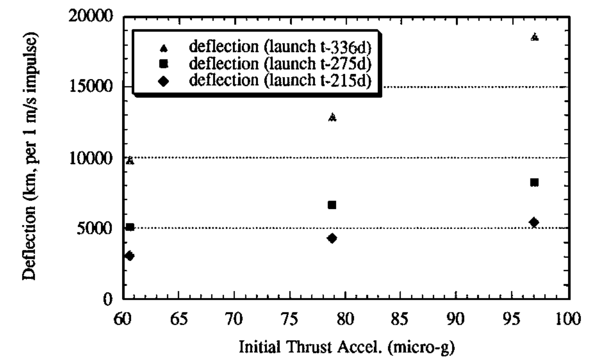

Fig. 3 Maximum specific deflection as a function of launch date and engine thrust acceleration.

Figure 2 shows the maximum amount of deflection that can be obtained, at what would otherwise be the time of close approach to the Earth, as a function of the interval between interception and close approach (on 19 September 1998). The impulse is assumed to be applied to the asteroid in the direction chosen, as described in the preceding section, to maximize the deflection at the subsequent close approach. Figure 2 shows that, if the asteroid is reached several months before the time of collision, each 1 m/s of velocity change imparted to the asteroid may yield a deflection distance comparable to the width of the Earth.

The maximum deflection for a given impulse applied to the asteroid, as a function of launch date and initial thrust acceleration, is shown in Fig. 3. Interception of the asteroid is accomplished using a minimum-flight-time trajectory, determined through the method described in the companion paper,[4] in which it has been assumed that the specific impulse of the low-thrust motor is $I_{\text{sp}} = 4000$ s and that the spacecraft departs Earth with a hyperbolic excess velocity $V_{\infty/E} = 2.94$ km/s. Note from Fig. 3 that, at any given thrust acceleration level, the deflection is increased for an earlier launch (and corresponding earlier arrival). Similarly, for a given launch date, the deflection increases with engine thrust acceleration level because higher thrust enables the spacecraft to intercept the asteroid more quickly. It is thus always the case, for this example where the close approach is imminent, that is, will occur before the asteroid makes a complete revolution about the sun, that an earlier interception, combined with a deflecting impulse directed optimally as defined in the preceding section, always yields a larger deflection. This seems intuitively correct and agrees with the results of Park and Ross[5] (cf. their Fig. 3) for the case of impulse time less than one period. It is clear from Fig. 3 that the improvement in deflection distance obtained from increasing the engine thrust and, hence, decreasing the time of flight is significant, for example, increasing the thrust acceleration by approximately 60%, from 61 to 97 μg, increases the deflection obtained by almost 100%.

The direction in which the optimal deflection impulse is to be applied is shown in Table 1 as a function of the time interval between interception and close approach to the Earth (on 19 September

Table 1 Direction of optimal deflection impulse applied at interception

Interval between interception and close approach, days	$\hat{e}_R$	$\hat{e}_T$	$\hat{e}_N$
19	0.983	0.184	$<10^{-3}$
30	0.970	0.243	$<10^{-3}$
44	0.954	0.300	$<10^{-3}$
52	0.945	0.327	$<10^{-3}$
65	0.934	0.357	$<10^{-3}$
72	0.923	0.385	$<10^{-3}$
76	0.900	0.435	$<10^{-3}$
86	0.959	0.281	$<10^{-3}$
104	0.920	0.391	$<10^{-3}$
134	0.932	0.362	$<10^{-3}$

1998). The direction may be inferred from the magnitude of the components of a unit vector in the direction of the deflecting impulse $\delta\bar{v}_0$. The unit vectors are those of an asteroid-fixed radial, tangential, normal (to asteroid orbit plane) basis, shown in Fig. 1. For this example, the optimal deflecting impulse is mostly radial, but may have a significant tangential component. The deflection normal to the asteroid plane is always negligibly small. There is not a monotonic variation of the direction as time progresses, and this is probably because the direction is expressed on a moving basis.

Conclusions

Many of the strategies for amelioration of the danger of an asteroid's collision with the Earth involve, at the time of interception, applying a small impulsive velocity change to the asteroid. In this work we show how, using the asteroid orbit state transition matrix, this impulse should be applied to maximize the deflection of the asteroid at the time of close approach. The algorithm requires no explicit (numerical) optimization and is, thus, easily applied. We also find, for the case in which the asteroid will make less than one complete orbit of the sun before its close approach or impact with the Earth, that the deflection resulting from a given magnitude of impulse increases the earlier the asteroid is intercepted. It is, thus, legitimate for this case to optimize the interception trajectory, minimizing the flight time, and then optimize the direction of the deflecting impulse separately, to maximize the deflection of the asteroid at what would otherwise be the impact time.

One important result, illustrated in Fig. 2 for a very representative ECA, is that, if the interceptor spacecraft is launched less than a year in advance of the asteroid's possible collision with Earth, the deflection obtained will be on the order of an Earth diameter for every 1 m/s velocity change applied to the asteroid. When it is considered how many millions of kilograms of mass some of the ECA possess, it would be very difficult to change their velocity by even 1 m/s. Thus, it may only be feasible to deflect asteroids of moderate size if they can be reached several years in advance of a potential collision.

References

[1]"List of the Potentially Hazardous Asteroids (PHAs)," URL: http://cfa-www.harvard.edu/iau/lists/Dangerous.html [cited 31 July 2001].

[2]Ahrens, T. J., and Harris, A. W., "Deflection and Fragmentation of Near-Earth Asteroids," *Hazards Due to Comets and Asteroids*, edited by T. Gehrels, Univ. of Arizona Press, Tucson, AZ, 1994, pp. 897–928.

[3]Solem, J. C., and Snell, C. M., "Terminal Intercept for Less Than One Period Warning," *Hazards Due to Comets and Asteroids*, edited by T. Gehrels, Univ. of Arizona Press, Tucson, AZ, 1994, pp. 1013–1034.

[4]Conway, B. A., "Optimal Low-Thrust Interception of Earth-Crossing Asteroids," *Journal of Guidance, Control, and Dynamics,* Vol. 20, No. 5, 1997, pp. 995–1002.

[5]Park, S. Y., and Ross, I. M., Two-Body Optimization for Deflecting Earth-Crossing Asteroids, *Journal of Guidance, Control, and Dynamics,* Vol. 22, No. 3, 1999, pp. 415–420.

[6]Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics,* AIAA Education Series, AIAA, Washington, DC, 1994, pp. 463–467.

Rate Feedback Control of Free-Free Uniform Flexible Rod

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I. Introduction

STATIC rate feedback is one of the simplest yet effective ways of controlling flexible structures. Its main purpose is to add damping, thus dissipating energy from the system, which is a key issue in vibration control. In servo systems the static rate feedback is sometimes used as an inner loop, providing a better-shaped "plant" for the outer loop, which enforces tracking.

It is well known[1] that a collocated structure with rate feedback is positive, that is, energy dissipating. Hence the system is stable, or asymptotically stable, if no rigid-body modes exist, regardless of the feedback gain value. However, one would like to find out what is the best, in some sense, gain. In certain cases collocated control may not be feasible. In others noncollocated control can be used intentionally because the area where vibration suppression is required and the actuation location do not coincide. Hence the properties of a system with noncollocated control are also of interest.

Infinite dimensional systems can be analyzed either in the frequency domain by using Nyquist stability criterion and similar tools[2] or by studying the poles pattern. References 3 and 4 contain an analysis of the pole location of flexible mechanical systems under noncollocated control. In this Note a similar approach is used for systems governed by the wave equation with static rate feedback. The relatively simple structure of such systems allows global results regarding the stability and the general shape of the poles map.

II. Transfer Function Derivation and Analysis

Consider the free-free uniform rod of length L subjected to torque moment $M(t)$ at one end, as shown in Fig. 1. $\theta(x,t)$ is a torsion angle at distance x from the forced end. The system is governed by the following wave equation and boundary conditions:

$$\frac{\partial^2\theta(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\theta(x,t)}{\partial t^2} \tag{1}$$

$$GI_p\frac{\partial\theta(x,t)}{\partial x}\bigg|_{x=0} = -M(t), \qquad \frac{\partial\theta(x,t)}{\partial x}\bigg|_{x=L} = 0 \tag{2}$$

where I_p denotes the polar moment of inertia, G is the shear elasticity modulus, ρ is the material density, and $c = (G/\rho)^{1/2}$ is the wave propagation velocity. Laplace transform with respect to time converts the partial differential Eq. (1) into an ordinary one in x:

$$\frac{\partial^2\theta(x,s)}{\partial x^2} - \frac{s^2}{c^2}\theta(x,s) = 0 \tag{3}$$

The solution of Eq. (3) is

$$\theta(x,s) = C_1(s)\exp(sx/c) + C_2(s)\exp[-(sx)/c] \tag{4}$$

$C_1(s)$ and $C_2(s)$ can be calculated from the boundary conditions. Introducing the normalized coordinate $\beta = x/L$ and the time constant $\tau = L/c$, the following transfer function is obtained:

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